On rotating regular nonabelian solutions

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Abstract

A general relation for the total angular momentum of a regular solution of the Einstein-Yang-Mills-Higgs equations is derived. Two different physical configurations, rotating dyons and rotating magnetic dipoles are discussed as particular cases. The issue of rotating pure Einstein-Yang-Mills regular solutions is addressed as well. Based on the results, we conjecture the absence of rotating regular solitons with a net magnetic charge.

1 INTRODUCTION

Extended objects such as non-Abelian monopoles and dyons provide a fertile ground for the study of the interplay between gravitational and gauge interactions in the presence of a spontaneously broken symmetry. Several results show conclusively that the gravitational interaction, although weak, can have significant effects on the properties of these objects and therefore, cannot be safely ignored (for a review and references see [1]). Most studies assume a spherical symmetry for the configurations, since the equations of motion reduce to coupled ordinary differential equations, that can be solved relatively easy. An interesting question is whether such solutions can be generalized to rotating ones. As the equations in this case are partial differential equations, results are much harder to find. A naive approach is to use perturbation theory to find slowly rotating solutions. However the absence or presence of rotating perturbative solutions, though indicative, is not in general conclusive to establish the absence or presence of exact solutions. For instance in boson star models slowly rotating perturbative solutions are absent, but solutions with a quantized angular momentum exist [2]. On the other hand in the Einstein-Yang-Mills (EYM) theory slowly rotating solutions are known to exist [3], but it is not clear whether they can be extended to exact solutions. It is therefore important to identify non-perturbative criteria to settle such questions.

Rotating solutions have been looked for in similar, but slightly different systems, which should be distinguished. The first are Yang-Mills-Higgs (YMH) regular solutions, the second black hole solutions, the third regular pure EYM solutions.

In the seminal paper of Julia and Zee [4], a profound connection between the angular momentum and the electric charge in a YMH theory has been suggested. However, in the absence of gravity, it has been shown that Julia-Zee dyons do not admit slowly rotating excitations [5]. Nonexistence results have been found also for slowly rotating regular solutions of EYM theory with bosonic matter [6]. In particular these results apply to the 't Hooft-Polyakov monopole and its self-gravitating generalizations.

The situation changes however when one allows for the presence of an event horizon. Rotating generalizations of static black hole solutions do exist. They necessarily have an electric charge. In the pure EYM case it was shown by Volkov and Straumann [7] that, within a perturbative approach, electrically charged black holes may exist. They are prohibited in the static spherically symmetric case by no-go theorems

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[8]. Therefore they are non-static and axially symmetric (see also [3]). These rotating black hole solutions have recently been constructed numerically by Kleihaus and Kunz [9]. The configurations possess two independent parameters: an angular momentum J and an electric charge Q. They are the EYM analogues of the Kerr-Newman solution in Einstein-Maxwell theory.

In the regular case, numerical solutions to the full stationary problem are still lacking. Regular rotating EYM solutions however were obtained perturbatively [3]. Here the angular momentum and the electric charge were found to be related. The existence of perturbative rotating solutions came as somewhat of a surprise, given the experience with other solitonic solutions. It was conjectured that this possibility is due to the masslessness of the fields leading to a powerlike behaviour at infinity. This emphasizes the importance of the asymptotics at infinity for the possible existence of exact solutions.

The aim of this paper is to present a non-perturbative approach to these problems and to argue that a rotating dyon necessarily contains an event horizon. This fact appears to be connected with the topological properties of the configurations (the existence of a net magnetic charge), as monopole-antimonopole regular configurations with electric charge and nonzero angular momentum exist. Of particular interest are the results in the pure EYM case. Concerning the rotating case, we shall demonstrate the general validity of the relation $J \propto Q$, which was found within a perturbative approach in [3]. However, the required asymptotic behavior of the gauge potentials appears to be incompatible with a nonzero value of the electric field at infinity. Thus, we conjecture the absence of rotating regular EYM solutions.

2   GENERAL FRAMEWORK AND BASIC EQUATIONS

We follow most of the notations and conventions used by Kleihaus and Kunz in their works [9, 10, 11]. The action for a self-gravitating non-Abelian $SU(2)$ gauge field coupled to a triplet Higgs field with the usual potential $V(\Phi)$, is (with G=c=g=1)

$$S = \int d^4x \sqrt{-g} [\frac{\mathcal{R}}{16\pi} - Tr\{\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + D_\mu\Phi D^\mu\Phi + V(\Phi)\}], \tag{1}$$

where

$$\begin{aligned} D_\mu &= \nabla_\mu + i[A_\mu,\ ], \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]. \end{aligned} \tag{2}$$

Varying the action (1) with respect to A_μ, Φ and $g_{\mu\nu}$, respectively, we have the equations

$$D_\mu F^{\mu\nu} = i[\Phi, D^\nu\Phi], \tag{3}$$

$$D_\mu D^\mu\Phi = i\frac{\delta V}{\delta\Phi}, \tag{4}$$

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{5}$$

where $G_{\mu\nu}$ is the Einstein tensor and the energy-momentum tensor has the form

$$T_{\mu\nu} = 2Tr\{F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + D_\mu\Phi D_\nu\Phi - \frac{1}{2}g_{\mu\nu}(D_\alpha\Phi D^\alpha\Phi + V(\Phi))\}. \tag{6}$$

Since we consider a stationary and axisymmetric system, the spacetime still has two Killing vector fields $\xi = \partial/\partial\varphi$ and $\psi = \partial/\partial t$ (also in the asymptotically flat regions of large r, r and θ are the usual spherical coordinates).

A metric form satisfying the circularity and Frobenius conditions [12] reads

$$ds^2 = -fdt^2 + \frac{m}{f}(dr^2 + r^2d\theta^2) + \frac{l}{f}r^2\sin^2\theta(d\varphi + \frac{\omega}{r}dt)^2, \tag{7}$$

where f, l, m and ω are functions on r and θ. This metric parametrization in terms of isotropic coordinates is useful when looking for numerical solutions (see e.g. [2, 9]).

Asymptotic flatness imposes on the metric functions the boundary conditions at infinity $f = m = l = 1$, $\omega = 0$. At the origin ($r = 0$), the boundary conditions on the metric functions read $\partial_r f = \partial_r m = \partial_r l = 0$, $\omega = 0$. For a configuration with parity reflection symmetry, the derivatives $\partial_\theta f, \partial_\theta m, \partial_\theta l$ and $\partial_\theta \omega$ vanish along the ρ- and z-axis (with $z = r\cos\theta$ and $\rho = r\sin\theta$). These boundary conditions apply whatever the matter content of the spacetime. Note that, as discussed in [13], a metric form (7) can be too restrictive for a general enough YM ansatz. However, in the next two sections, we don't use an explicit form of the metric.

The symmetry of the gauge field under a spacetime symmetry means that the action of an isometry can be compensated by a suitable gauge transformation [13, 14].

For the Killing vector ψ, we choose a gauge such that $\partial A/\partial t = 0$. However, a rotation around the $z-$axis can be compensated by a gauge rotation

$$\mathcal{L}_\xi A = D\Psi, \tag{8}$$

and therefore

$$\begin{aligned} F_{\mu\varphi} &= D_\mu W, \\ D_\varphi \Phi &= i[W, \Phi], \end{aligned} \tag{9}$$

where $W = A_\varphi - \Psi$.

3   ANGULAR MOMENTUM IN EYMH THEORY

The total angular momentum for a regular spacetime (*i.e.* no interior boundary) can be expressed as

$$J = \frac{1}{8\pi} \int R^\alpha{}_\beta \xi^\beta_{(\varphi)} d^3\Sigma_\alpha \tag{10}$$

and, from the Einstein equations

$$\begin{aligned} J &= \int T^t_\varphi \sqrt{-g} d^3 x \\ &= \int 2Tr\{F_{r\varphi}F^{rt} + F_{\theta\varphi}F^{\theta t} + D_\varphi \Phi D^t \Phi\}\sqrt{-g} d^3 x \end{aligned} \tag{11}$$

(equivalently, J can be read off from the asymptotic expansion of the metric tensor). We show that this volume integral can be converted into a surface integral in terms of matter fields. Using the potential W we find in (11)

$$\begin{aligned} T^t_\varphi \sqrt{-g} &= 2Tr\{(D_r W)F^{rt}\sqrt{-g} + (D_\theta W)F^{\theta t}\sqrt{-g}) + i[W, \Phi]D^t \Phi \sqrt{-g}\} \\ &= 2Tr\{D_r(WF^{rt}\sqrt{-g}) + D_\theta(WF^{\theta t}\sqrt{-g}) \\ &\quad - W(D_r F^{rt} + D_\theta F^{\theta t})\sqrt{-g} + i[W, \Phi]D^t \Phi \sqrt{-g}\}. \end{aligned} \tag{12}$$

As a consequence of the YM equations (3) we have also

$$D_r F^{rt} + D_\theta F^{\theta t} = -D_\varphi F^{\varphi t} + i[\Phi, D^t \Phi]. \tag{13}$$

Also, from the relations (9) we write

$$Tr\{WD_\varphi F^{\varphi t}\} = 0. \tag{14}$$

Making use of the fact that the trace of a commutator vanishes we obtain

$$T^t_\varphi = 2Tr\{\frac{1}{\sqrt{-g}}\partial_\mu(WF^{\mu t}\sqrt{-g})\}. \tag{15}$$

3

Thus, ignoring possible singularities the expression of the total angular momentum is

$$
\begin{aligned}
J &= \oint_\infty 2Tr\{WF^{\mu t}\}dS_\mu \\
&= -2\pi \lim_{r\to\infty} \int_0^\pi d\theta \sin\theta \; r^2 [W^{(r)}F_{rt}^{(r)} + W^{(\theta)}F_{rt}^{(\theta)} + W^{(\varphi)}F_{rt}^{(\varphi)}].
\end{aligned}
\tag{16}
$$

Observe that the above relation is a broader result, since it does not depend crucially on the particular form of the symmetry-breaking Higgs potential. In particular, (16) remains valid in the absence of the Higgs field. Also, this results holds in the flat spacetime limit, following the general definition (11) of the angular momentum. Note that we do not fix the gauge in the derivation of this relation.

The field equations (3-5) admit classical solutions, whose energy is finite and localized in small regions. Magnetic monopoles and dipoles are example of such configurations of gravitating vector and scalar fields, which can be stable as a result of their nontrivial topology. The size, the mass and other features of these configurations generally have to be abstracted from the solution of complicated nonlinear equations.

The extreme nonlinearity of the YMH equations even in flat spacetime means that they are only integrable in the Bogomol'nyi-Prasad-Sommerfield (BPS) limit. The equations in curved spacetime are still more complicated, and so the only realistic approach is to solve them numerically. However, in order to evaluate the total angular momentum for a regular spacetime we need the asymptotics of the gauge functions only. For asymptotically flat finite energy solutions and *a specific ansatz* these expressions can be read off from the field equations.

4   ROTATING DYONS SOLUTIONS

A general axially symmetric YMH ansatz has been considered for the first time by Manton [15], and has been generalized by Rebbi and Rossi [16] for winding number $n > 1$ when discussing multimonopole solutions. The usual ansatz used by various authors when discussing axially symmetric YMH configurations is derived from the ansatz of Rebbi and Rossi. In spherical coordinates it reads (see also [17])

$$
\begin{aligned}
A_r &= \frac{1}{r}H_1(r,\theta)\frac{u_\varphi}{2}, \\
A_\theta &= (1 - H_2(r,\theta))\frac{u_\varphi}{2}, \\
A_\varphi &= -n\sin\theta \left[H_3(r,\theta)\frac{u_r}{2} + (1 - H_4(r,\theta))\frac{u_\theta}{2} \right], \\
A_t &= \eta \left[H_5(r,\theta)\frac{u_r}{2} + H_6(r,\theta)\frac{u_\theta}{2} \right], \\
\Phi &= \eta \left[\phi_1(r,\theta)\frac{u_r}{2} + \phi_2(r,\theta)\frac{u_\theta}{2} \right],
\end{aligned}
\tag{17}
$$

where η is the asymptotic value of the Higgs field, with unit vectors

$$
\begin{aligned}
u_r &= (\sin\theta\cos n\varphi, \; \sin\theta\cos n\varphi, \; \cos\theta), \\
u_\theta &= (\cos\theta\cos n\varphi, \; \cos\theta\cos n\varphi, \; -\sin\theta), \\
u_\varphi &= (-\sin n\varphi, \; \cos n\varphi, \; 0).
\end{aligned}
\tag{18}
$$

The winding number n corresponds to the topological charge of the solutions [16, 18]. For $n = 1$ the ansatz reproduces the spherically symmetric dyons. The ansatz (17) is not the most general one, since it is obtained by imposing a further discrete symmetry $M_{xz} \bigotimes C$, where the first factor represents reflection through the xz-plane and the second factor denotes charge conjugation [19]. However, this is the most general dyon ansatz fulfilling the circularity condition. For this ansatz $\Psi = n\cos\theta\frac{u_r}{2} - n\sin\theta\frac{u_\theta}{2}$ and

$$
W = (-n\cos\theta - n\sin\theta H_3)\frac{u_r}{2} + n\sin\theta H_4\frac{u_\theta}{2}.
\tag{19}
$$

4

The boundary conditions at infinity consistent with the requirements of regularity, finite energy and symmetry, are [17]

$$H_i = 0, \ i = 1, 2, 3, 4, 6; \quad H_5 = \alpha; \quad \Phi_1 = 1; \quad \Phi_2 = 0. \tag{20}$$

(where $\alpha \leq 1$), and

$$H_i = 0, \ i = 1, 3, 5, 6; \quad H_i = 1, \ i = 2, 4; \quad \Phi_i = 0, \ i = 1, 2, \tag{21}$$

at the origin. Given the parity reflection symmetry, we need to consider solutions only in the region $0 \leq \theta \leq \pi/2$; on the z- and ρ-axis the functions H_1, H_3, H_6, Φ_2 and the derivatives $\partial_\theta H_2, \partial_\theta H_4, \partial_\theta H_5$ and $\partial_\theta \Phi_1$ are to vanish.

In our units, the magnetic charge of the dyon is n. Also, when using the electromagnetic 't Hooft field strength tensor we have the expression for the electric charge [17]

$$Q = \lim_{r \to \infty} 4\pi\eta r^2 \partial_r H_5. \tag{22}$$

The nongravitating axially symmetric dyon solutions of Ref. [17] have been found within this ansatz. As far as we know, their curved spacetime generalization is still missing.

By using the ansatz (17) we find

$$\lim_{r \to \infty} Tr(r^2 W F_{rt}) = -\frac{nQ}{8\pi} \cos\theta \tag{23}$$

and therefore $J = 0$. Thus, an axially symmetric regular dyon has a vanishing total angular momentum.

We mention also a different attempt to generate (flat-space) axially symmetric YMH solutions with a nonzero angular momentum (see [20]). In that approach, the Bogomol'nyi equations are casted in the form of the Einstein equations and known exact solutions of general relativity are used to generate new YMH solutions. However, unlike the Kerr solutions, the corresponding nonabelian configurations do not carry any intrinsic angular momentum [20].

However, the dyon solutions do not exhaust the configurations allowed by the EYMH system. One may ask whether this result remains valid for a different physical situation, in the absence of a net magnetic charge.

5 ROTATING MONOPOLE-ANTIMONOPOLE SOLUTIONS

As shown by Taubes [21], there are smooth, finite action solutions to the SU(2) YMH equations in the BPS limit, which do not satisfy the first order Bogomol'nyi equations. A monopole-antimonopole pair (MAP) bound state (carrying zero net magnetic charge) and possessing only axial symmetry corresponds to such a non-BPS solution [22, 23]. While dyon solutions reside in topologically nontrivial sectors, the MAP pair solution is topologically trivial; in particular it is unstable.

When gravity is coupled to YMH theory, regular MAP solutions and black holes with magnetic dipole hair solutions have been found numerically [24]. In the flat spacetime limit, the existence of multi-MAP configurations with a net electric charge has been predicted in [17]. The resulting solutions possess magnetic charges of opposite sign, but electric charges of equal sign. It is natural to expect that these flat spacetime solutions can be generalized to curved spacetime.

A specific MAP ansatz (consistent also with the circularity condition) used in numerical calculations [24] suplemented with a nonvanishing time component of the gauge field reads

$$
\begin{aligned}
A_r &= \frac{1}{2r} H_1(r, \theta) \frac{u_\varphi}{2}, \\
A_\theta &= 2(1 - H_2(r, \theta)) \frac{u_\varphi}{2}, \\
A_\varphi &= -2\sin\theta \left[H_3(r, \theta) \frac{u_r}{2} + (1 - H_4(r, \theta)) \frac{u_\theta}{2} \right],
\end{aligned}
\tag{24}
$$

$$A_t = \eta \left[H_5(r,\theta)\frac{u_r}{2} + H_6(r,\theta)\frac{u_\theta}{2} \right],$$

$$\Phi = \eta \left[\phi_1(r,\theta)\frac{u_r}{2} + \phi_2(r,\theta)\frac{u_\theta}{2} \right],$$

where, this time

$$
\begin{aligned}
u_r &= (\sin 2\theta \cos\varphi, \ \sin 2\theta \cos\varphi, \ \cos 2\theta), \\
u_\theta &= (\cos 2\theta \cos\varphi, \ \cos 2\theta \cos\varphi, \ -\sin 2\theta), \\
u_\varphi &= (-\sin\varphi, \ \cos\varphi, \ 0)
\end{aligned}
\tag{25}
$$

(this ansatz can also be extended to include a winding number n [23]).

The finite energy, fundamental gravitating MAP solution with a net electric charge will satisfy a different set of boundary conditions at infinity

$$H_1 = H_2 = 0, \quad H_3 = \sin\theta, \quad 1 - H_4 = \cos\theta, \quad H_5 = \alpha, \quad H_6 = 0, \quad \Phi_1 = 1, \quad \Phi_2 = 0, \tag{26}$$

(where $\alpha \leq 1$). We impose at the origin the boundary conditions

$$H_1 = H_3 = H_2 - 1 = H_4 - 1 = 0 ,$$
$$\sin 2\theta \Phi_1 + \cos 2\theta \Phi_2 = 0 , \quad \partial_r (\cos 2\theta \Phi_1 - \sin 2\theta \Phi_2) = 0 ,$$
$$\sin 2\theta H_5 + \cos 2\theta H_6 = 0 , \quad \partial_r (\cos 2\theta H_5 - \sin 2\theta H_6) = 0.$$

On the z-axis the functions H_1, H_3, H_6, Φ_2 and the derivatives $\partial_\theta H_2, \partial_\theta H_4, \partial_\theta H_5, \partial_\theta \Phi_1$ have to vanish, while on the ρ-axis the functions $H_1, 1 - H_4, H_6, \Phi_2$ and the derivatives $\partial_\theta H_2, \partial_\theta H_3, \partial_\theta H_5, \partial_\theta \Phi_1$ have to vanish.

For this ansatz $\Psi = \cos 2\theta \frac{u_r}{2} - \sin 2\theta \frac{u_\theta}{2}$ and

$$W = (-2\sin\theta H_3 - \cos 2\theta)\frac{u_r}{2} + (\sin 2\theta - 2\sin\theta(1 - H_4))\frac{u_\theta}{2}. \tag{27}$$

From (16) we find a relation between the angular momentum and the electric charge (defined by (22))

$$J/Q = 1. \tag{28}$$

This connects the quantization of charge and angular momentum. A regular magnetic dipole cannot rotate unless it is endowed with a net electric charge. At this stage we cannot say anything about the magnitude of the magnetic dipole moment and the gyromagnetic ratio of the configuration. This is clearly an interesting subject but further numerical work is required.

6 ROTATING EYM REGULAR SOLUTIONS

An important particular case of the general relation (11) corresponds to the absence of a Higgs field. In the black hole case, Kleihaus and Kunz were recently able to find rotating black holes EYM solutions within a nonperturbative approach [9]. Thus, one may ask whether there are pure EYM globally regular solutions with a nonzero angular momentum. As proven above, this fact is related to the existence of an electric YM potential. For the spherically symmetric case, there are no-go theorems forbidding these solutions [8]. For the stationary, axisymmetric case, some contradictory results have been obtained.

In Ref. [25], based on some general topological considerations, it has been conjectured that the Bartnik-McKinnon solutions are the only stationary nonsingular solutions of the EYM equations. However, a perturbative analysis based on the assumption of linearization stability has been carried out with a different result. The authors of Ref. [3] have assumed the existence of a family of stationary regular solutions of the EYM equations, approaching the static solutions for zero angular momentum. The tangent to this family at $J = 0$ satisfies the linearized EYM equations. The results of their perturbative study imply

that all static spherically symmetric solutions admit slowly rotating excitations with continuous angular momentum J and YM electric charge Q proportional to J. Conversely, it is reasonable to expect that for a well-behaved solution of the linearized equations around the static configurations there will be an exact family of rotating solutions.

As far as we know, no results are available for the regular case, excepting some preliminary considerations in [26].

A suitable form of the YM connection in this case is

$$
\begin{aligned}
A_r &= \quad \frac{1}{r} H_1(r,\theta) \frac{u_\varphi}{2}, \\
A_\theta &= \quad (1 - H_2(r,\theta)) \frac{u_\varphi}{2}, \\
A_\varphi &= \quad -n\sin\theta \left[H_3(r,\theta) \frac{u_r}{2} + (1 - H_4(r,\theta)) \frac{u_\theta}{2} \right], \\
A_t &= \quad \left[H_5(r,\theta) \frac{u_r}{2} + H_6(r,\theta) \frac{u_\theta}{2} \right].
\end{aligned}
\tag{29}
$$

The unit vectors u_a are given by (18); here also

$$
W = (-n\cos\theta - n\sin\theta H_3) \frac{u_r}{2} + n\sin\theta H_4 \frac{u_\theta}{2}
\tag{30}
$$

The boundary conditions for the magnetic potentials, familiar from the static configurations [10], read

$$
H_1 = H_3 = 0, \quad H_2 = H_4 = (-1)^k,
\tag{31}
$$

at infinity, and

$$
H_1 = H_3 = 0, \quad H_2 = H_4 = 1,
\tag{32}
$$

at the origin. Here k is the node number of the H_2 and H_4 functions (note the vanishing of the magnetic charge). The asymptotic expansion of electric potentials is

$$
H_5 \sim (V - \frac{Q}{r})\cos\theta, \quad H_6 \sim (-1)^{k+1}(V - \frac{Q}{r})\sin\theta.
\tag{33}
$$

Also, for $r = 0$ we have

$$
\partial_r H_5 = \partial_r H_6 = 0.
\tag{34}
$$

The boundary conditions on the z-axis are $H_1 = H_3 = H_6 = 0$, $\partial_\theta H_2 = \partial_\theta H_4 = \partial_\theta H_5 = 0$, and agree with the boundary conditions on the ρ-axis, except for $H_5 = 0$, $\partial_\theta H_6 = 0$.

For the above set of boundary conditions and the asymptotic expansion (33), we obtain from (16) the relation

$$
J/Q = 4\pi n.
\tag{35}
$$

Thus, as already obtained within the perturbative approach [3], rotating stationary Einstein-Yang-Mills solitons are necessarily electrically charged.

A nonzero asymptotic magnitude of A_t is crucial in obtaining this result. By combining the existence of an electric potential (*i.e.* $F_{\mu t} = D_\mu A_t$) and using the Yang-Mills equations we find that the electric term in the YM energy can be converted into a boundary integral and (see [25])

$$
VQ = 2Tr(\int F_{\mu t} F^{\mu t} \sqrt{-g} d^3 x).
\tag{36}
$$

Thus a vanishing electric potential at infinity implies no electric field at all. In the Abelian theory it is possible to gauge V away; in the non-Abelian theory, such a gauge transformation would render the whole configuration time-dependent.

Stimulated by the results obtained within the perturbative approach, we have initiated a numerical study of the problem. Our methods were similar to those used by Kleihaus and Kunz in their works [10, 9]. Their scheme solves the field equations following an iteration procedure. One starts with a known static configuration and increases the value of V in small steps. The field equations are discretized on a nonequidistant grid and the resulting system is solved iteratively until convergence is achieved. The numerical calculations are performed by using the program FIDISOL, based on the iterative Newton-Raphson method [27]. To fix the gauge, we choose the usual gauge condition [9] $r\partial_r H_1 - \partial_\theta H_2 = 0$.

The numerical results we found are the following. First (for $n = 1, 2, ...$), we have found charged, rotating solutions for small values of V only, typically up to the order 10^{-3} (the numerical errors increase with the value of V and depends also on the order of consistency of the differential formulae for the derivatives). For higher V, the numerical iteration fails to converge. Also, there are only small differences between the total energy of these rotating configurations and those of the corresponding static solutions. Secondly, we have proven numerically the validity of the relation (35).

We suspect the source of this behavior to reside in the long range behavior of the magnetic potentials H_i. For static configurations, the asymptotic solutions are familiar from Ref. [28]. However, it seems that even a small V will alter these asymptotics. For instance, as $r \to \infty$ the equation for the magnetic gauge potential H_2

$$
\begin{aligned}
0 = & \ H_{2,r,r} + \frac{H_{2,\theta,\theta}}{r^2} - \frac{H_{1,\theta}}{r^2} + \ln\left(\frac{f\sqrt{l}}{m}\right)_{,r}(H_{2,r} + \frac{H_{1,\theta}}{r}) + \frac{1}{r^2}\ln\left(\frac{f\sqrt{l}}{m}\right)_{,\theta}(H_{2,\theta} - rH_{1,r}) \\
& + \frac{\cot\theta}{r^2}(H_{2,\theta} - rH_{1,r}) - \frac{n^2 m}{lr^2}\left(1 - \frac{\omega^2 l \sin^2\theta}{f^2}\right)\left(H_4 H_{3,\theta} - H_3 H_{4,\theta} + H_2(H_3^2 + H_4^2 - 1)\right) \\
& + \frac{H_2 - H_4}{\sin^2\theta} + \cot\theta(2H_2 H_3 - H_{4,\theta})\right) + \frac{m}{f^2}\left(H_2(H_6^2 + H_5^2) + H_5 H_{6,\theta} - H_6 H_{5,\theta}\right) \\
& - \frac{\omega m}{rf^2}n\sin\theta\Big(H_5(H_{4,\theta} - 2H_2 H_3 - \cot\theta(H_2 - H_4) + H_6(H_{3,\theta} - 1 + 2H_2 H_4 + \cot\theta H_3) \\
& - \cot\theta(H_{6,\theta} + H_2 H_5) - H_3 H_{6,\theta} - H_4 H_{5,\theta}\Big)
\end{aligned}
$$

(37)

takes the linearized form

$$
\begin{aligned}
h_{2,r,r} \ + & \ \frac{h_{2,\theta\theta}}{r^2} - \frac{H_{1,\theta}}{r^2} - \frac{\cot\theta}{r^2}(rH_{1,r} - h_{2,\theta}) - \frac{n^2}{r^2}\Big((-1)^k H_{3,\theta} + h_2 + h_4 + (-1)^k 2\cot\theta H_3 \\
& + \cot^2\theta(h_2 - h_4)\Big) + V^2 h_2 = 0
\end{aligned}
$$

(38)

Here $H_2 = (-1)^k + h_2$, $H_4 = (-1)^k + h_4$; h_2, h_4 are small deviations and must decay asymptotically to zero. For large enough r, the last term in (38) may not be regarded as a "small perturbation". Thus, it seems that the term $V^2 h_2$ implies oscillatory behavior of the h_2 rather than an exponential or polynomial decay. Given these facts, we interpret the numerical results as follows. For very small V the $V^2 h_2$ term does not show up in the numerics. The system therefore mimics the linearized theory. This is consistent with the quantization of J/Q. For larger values of V no convergence is found anymore, since the solutions do not fall off at infinity.

A similar approch can be applied for the full system. With the asymptotic values (31, 33), the reduced system linearizes asymptotically and we end up with the problem of finding the asymptotic behavior of a perturbed linear two-dimensional system of equations. For all equations, the A_t components of the gauge field act like an isotriplet Higgs field with negative metric, and by themselves cause the magnetic components to oscillate rather than decrease exponentially as $r \to \infty$.

This fact cannot be obtained within the perturbative approach used in [3], since the terms on the form A_t^2 are ignored in the first order of perturbation theory. Thus, if we insist on the boundary conditions (31), $V = 0$ is required and, from (36) we obtain a purely magnetic nonrotating configuration. We notice that a related observation has been made in [9], where, in the limit of vanishing horizon radius, no global regular solutions are obtained for $V = 0$.

8

The existence of regular rotating generalizations of the Bartnik-McKinnon particles and their higher winding number generalization therefore is highly improbable. The appearance of oscillating solutions in the perturbative expansion is a strong indication, that such solutions would not be localized. For an exact mathematical proof a much more detailed analysis of the asymptotic solutions would be necessary. Given the experience with the static EYM axially symmetric solutions [28, 10], this appears to be prohibitively complicated. For instance, non-analytic terms in $1/r$ have to be considered and a separate analysis is required for every n.

7   FURTHER DISCUSSIONS

The essence of our results is a new expression for the Komar angular momentum in a EYMH theory as a surface integral in terms of YM fields. We have shown that this expression is a useful tool for the issue of rotating nonabelian solutions.

We point out that many extensions are possible. The inclusion of other fields coupled to gauge fields, such as a dilaton or a doublet Higgs field is possible in the framework of our derivation of (16). In this way, we found that the results of [5, 6] remain valid in the nonperturbative approach. An axially symmetric dyon configuration necessarily possesses a vanishing total angular momentum. However, for a different EYMH configuration, consisting of a pair monopole-antimonopole we have predicted that the total angular momentum is proportional to the electric charge. These results suggest a connection with the topological properties of the configurations (the existence of a net magnetic charge).

Of course further refinements in the analysis are possible. For instance, in the derivation of (16), we assumed regularity of the quantity $WF^{\mu t}$. Also, the ansätze we have used are not apriori well defined on the z- axis and at the origin; the question of regularity for the gauge potentials is a very intricate one and separate analysis are required for every case (see $e.g.$ [28, 29]).

The case of rotating regular EYM solitons is not completely settled, but we have presented strong arguments implying the absence of rotating regular configurations. The further investigation of the asymptotic behavior of the gauge functions would be desirable. Furthermore, a study of the EYM system beyond the circular sector of the theory is of interest.

When coupled to gravity, the flat spacetime solitons admit also "coloured" black hole generalizations. We can use the above approach to derive the total angular momentum expression (within a specific YMH ansatz) for a rotating dyonic black hole and for a rotating black hole with magnetic dipole hair. The boundary conditions at infinity are still valid, while a new set of conditions are to be imposed on the event horizon. For a rotating dyonic black hole we find this time a nonvanishing J, resulting from the event horizon contribution. Given the inner boundary contribution, the relation (28) is no longer valid for a rotating black hole with magnetic dipole hair. The angular momentum and electric charge are independent quantities.

The above arguments have been made within a YMH ansatz satisfying the circularity condition and the proof is therefore not complete for the general case. However this ansatz is rather general and since the arguments are based on conservation of angular momentum and the behaviour at infinity of the fields we expect the results to be valid beyond the specific ansatz within which they are derived. We expect also to obtain a vanishing total angular momentum for a regular configuration consisting of a different number of monopoles and antimonopoles.

We therefore put foreward the following conjecture: *"For any regular solution in a gauge theory coupled to gravity, a nonvanishing total angular momentum is incompatible with a net magnetic charge. Any dyon solution with a nonzero angular momentum necessarily contains an event horizon."*

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